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THESIS

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A NONLINEAR PROGRAMING MODEL FOR OPTIMIZED SORTIE ALLOCATION

by

Klaus Paul Wirths

March 1989

Thesis Advisor: Alan R.Washburn

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The United States Air Force uses a nonlinear programming model to assess the utilization of weapons and sorties needed achieve a maximum value of destroyed targets in a multi-period, Theater-Level conflict. The current model is modified by nstraining the consumption of weapons. Alternate objective functions are introduced. Their meaning and influence on the ptimization is compared. An increase in the worth of destroyed targets is gained if the model can more flexibly utilize eapons than is currently the case. The optimization can be further improved if all time periods are considered simultane-usly while assigning sorties to targets, rather than the current myopic approach.

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A Nonlinear Programing Model for Optimized Sortie Allocation

by

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requirements for the degree of

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ABSTRACT

The United States Air Force uses a nonlinear programming model to assess the utilization of weapons and sorties needed to achieve a maximum value of destroyed targets in a multi-period, Theater-Level conflict. The current model is modified by constraining the consumption of weapons. Alternate objective functions are introduced. Their meaning and influence on the optimization is compared. An increase in the worth of destroyed targets is gained if the model can more flexibly utilize weapons than is currently the case. The optimization can be further improved if all time periods are considered simultaneously while assigning sorties to targets, rather than the current myopic approach.

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THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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I. INTRODUCTION

In 1988 the United States Air Force purchased over $ 2 billion worth of weapons for use in different theaters around the world. The projected need for the quantity of different weapon types is based on an annual Nonnuclear-Weapon Consumables Analysis (NCAA) performed by the Directorate of Plans, USAF [Ref. 1]. Unlike other services, the USAF relies widely on mathematical programming models in order to optimize the allocation of weapons.

In 1974 RAND developed a nonlinear programming model that optimizes the number of different sortie types assigned to several target types by maximizing the military worth of killed targets [Ref. 2: p. 5]. Since each target type was given a different target value, the model attempts to assign sorties to maximum value targets first. To avoid an undesired concentration of sortie allocations to a few or even one target type, a nonlinear objective function was introduced. Within the model only the number of available targets and sorties are constrained. The expenditure of weapons is not considered. The number of targets one sortie is able to destroy is expressed by an effectiveness parameter that depends only on sortie and target type.

The required input data structure for the RAND-model is a simplification of the much more complex data base contained in the Joint Munitions Effectiveness Manual (JMEM) used by USAF. The JMEM data base determines effectiveness as a function of weather and mission profile (tactic) as well as type of aircraft and type of target. In the current operation a model called SELECTOR sorts the JMEM data base so that for each sortie-target type combination, all feasible tactics are ordered from the most to the least cost-effective, including the cost of aircraft attrition. This list is referred to as the Preferred Weapon List.

The data in the Preferred Weapon List must be reduced to input parameters depending only on sortie and target type as mentioned earlier. This is basically done by selecting the most cost-effective tactic from the list feasible for weather situations considered in the model. After the optimization has determined the optimal number of sorties assigned to different targets, the number of remaining targets and the expenditure of weapons is evaluated. This process is repeated in subsequent time periods with a new inventory of sorties and also by recording the remaining number of active targets and weapons available. In this way, tactical changes in a given scenario over time are considered by optimizing sequentially for discrete time periods. This process is accomplished in one programming model and is called the HEAVY ATTACK model. The USAF interest is mainly in the consumption of weapons utilized over all time periods.

The objectives of this Thesis are to include a weapons constraint in the RAND-model and to investigate alternatives to the currently used objective function. In addition, the RAND-model is expanded so that more available information is included in the optimization in order to gain a higher total military worth of killed targets than is currently achieved. Therefore, the consumption of weapons used by less cost-effective tactics is investigated when other weapons, used by the most cost-effective tactic, are exhausted. As a final consideration, one global optimization over all time periods is compared to the current sequential optimization method. Global optimization achieves a higher overall worth of killed targets. However, gaining a higher military worth of killed targets serves only as an aid in analyzing the predicted need of weapons. The value of the revisions suggested in this Thesis have to be measured on their ability to satisfy the demands of the USAF and simultaneously meet budget constraints.

II. BASIC STRUCTURE OF HEAVY ATTACK

A. THE ORIGINAL RAND - MODEL

In 1974 RAND developed a nonlinear programming model whose objective was to determine the optimal number of sorties of type i assigned to targets of type j by maximizing the total military value of destroyed targets. The relationship between an assigned sortie and a target kill is established by introducing "sortie effectiveness" $\overline{E}_{i,j}$. The parameter $\overline{E}_{i,j}$ defines the average number of kills that one sortie of type i will achieve when it is assigned to targets of type j.

Definition of index

i	sortie type
j	target type

Parameter

T_j	total number of type j targets available at the beginning of a time period
V_j	military worth of type j target
S_i	total number of type i sorties available
$\overline{E}_{i,j}$	average number of type j targets killed by one type i sortie

Variables

$SX_{i,j}$	number of type i sorties assigned to type j targets

Model

$$\text{Max} \quad z = \sum_{j} V_j \times f_j\left(\sum_{i} \overline{E}_{i,j} \times SX_{i,j}\right)$$

s.t.

$$\sum_{j} SX_{i,j} \leq S_i \qquad \forall\ i$$

$$f_j\left(\sum_{i} \overline{E}_{i,j} \times SX_{i,j}\right) \leq T_j \qquad \forall\ j$$

$$\sum_{j \in J} SX_{i,j} \leq c \times \sum_{j} SX_{i,j} \qquad \forall\ i$$

where J is a subset of all targets of type j and $0 < c < 1$.

$$0 \leq SX_{i,j} \qquad \forall\ i, j$$

$f_j(\sum_i \overline{E}_{i,j} \times SX_{i,j})$ is a concave function that approaches 1 for large arguments. The RAND - model (and HEAVY ATTACK) utilizes a specific analytic from that will be examined in detail later. The recipe constraints $\sum_{j \in J} SX_{i,j} \leq c \times \sum_{i} SX_{i,j}$ limit the number of sorties of type i which are assigned to a list of targets by a fraction of the total number of sorties of type i. Since these constraints are not used by the USAF in their current weapon analysis, this inequality will omitted from now on in the Thesis.

B. THE ROLE OF SELECTOR

Based on the information contained in the JMEM the effectiveness of a sortie depends on sortie type, target type, weapon type , weather and tactics or mission profile.

Definition of index

i	sortie type
j	target type
k	weapon type
w	weatherband index
r	index for used tactic

Definition of parameter

$E_{i,j,r,w}$	number of type j targets killed by one type i sortie using tactic r in weatherband w
$B_{i,j,r,w}$	number of weapons carried by one type i sortie which is assigned to type j target in weatherband w and using tactic r
$K_{i,j,r,w}$	type of weapon which is loaded on sortie i and will be deployed to target j by using tactic r in weatherband w

The JMEM data have too many subscripts to match the required input data structure of the RAND - model. The number of subscripts of a sortie needs to be reduced so that $E_{i,j}$depends only on sortie and target type. The first part of the task of reducing the number of subscripts from 4 to 2 is accomplished by the sorting program SELECTOR. The output data of SELECTOR - referred to as Preferred Weapon List - contains for each different sortie - target type combination five distinct items:

1. The worst weatherband in which a tactic can be used.
2. The types of weapons that can be allocated.
3. The relative cost-efficiency of a tactic given by its order on the list.
4. The number of targets which can be killed by one sortie.
5. The number of weapons that can be carried by one sortie for each weapon type (mixes of weapons are not considered).

The data structure of the Preferred Weapon List, which will be used later for the aggregation of the input data $\overline{E}_{i,j}$ for the RAND - model, is illustrated by the following example:

Subset of data from Preferred Weapon List

i	j	r	w	$K_{i,j,r,w}$	$E_{i,j,r,w}$	$B_{i,j,r,w}$
1	29	1	4	3	0.137	4
1	29	2	3	1	0.664	6
1	29	3	2	17	1.580	2
1	29	4	5	17	1.600	2

For example, the most cost-efficient and feasible tactic for weatherband w = 3 is tactic r = 2. Tactic r = 1 is more cost-efficient because it is first on the list, but is only feasible in weatherband w = 4 or higher. Weatherband w = 1 expresses best weather while weatherband w = 6 represents the worst weather. Tactic r = 3 is feasible (a tactic feasible in w is always feasible in better weatherbands) but less cost-efficient than tactic r = 2.

The given data can be represented in the following way:

Table 1. $E_{I,J,R,W}$ - VALUES: Number of targets of type j killed by one sortie of type i using tactic r in weatherband w.

i	j	r	w = 1	w = 2	w = 3	w = 4	w = 5	w = 6
1	29	1	0	0	0	0.137	0.137	0.137
1	29	2	0	0	0.664	0.664	0.664	0.664
1	29	3	0	1.580	1.580	1.580	1.580	1.580
1	29	4	0	0	0	0	1.600	1.600

Table 2. $B_{I,J,R,W}$ - VALUES: Number of weapons that are loaded on one sortie of type i which is assigned to target type j and using tactic r in weatherband w.

i	j	r	w=1	w=2	w=3	w=4	w=5	w=6
1	29	1	0	0	0	4	4	4
1	29	2	0	0	6	6	6	6
1	29	3	0	2	2	2	2	2
1	29	4	0	0	0	0	2	2

Table 3. $K_{I,J,R,W}$ - VALUES: Type of weapon that is allocated to a sortie of type i which is assigned to a target of type j and using tactic r in weatherband w

i	j	r	w=1	w=2	w=3	w=4	w=5	w=6
1	29	1	0	0	0	3	3	3
1	29	2	0	0	1	1	1	1
1	29	3	0	17	17	17	17	17
1	29	4	0	0	0	0	17	17

Since HEAVY ATTACK only considers the tactic at the top of the list for each weatherband, and since weapon type is implied by tactics. SELECTOR essentially reduces the number of subscripts from 4 to 3.

C. DETERMINATION OF $\overline{E}_{I,J}$ IN HEAVY ATTACK

An important assumption for HEAVY ATTACK in order to understand the logic behind the aggregation of $\overline{E}_{i,j}$ is that the weather is *not known* at the time when sorties are assigned to targets. This leads to the condition that the effectiveness of a sortie and the consumption of weapons in a particular weatherband has to be proportional to the probability that this weather will occur.

This probability is represented in HEAVY ATTACK by a given distribution of 6 distinct weatherbands:

PR_w = probability that weatherband w will occur at a certain time in the future.
w = 1, 2,...., 6.

Throughout this Thesis the following distribution is used:

Table 4. **WEATHER DISTRIBUTION IN HEAVY ATTACK:** Probability that weatherband w occurs when sorties are allocated to targets.

	w=1	w=2	w=3	w=4	w=5	w=6
PR_w	0	0.02	0.14	0.07	0.07	0.70

Since weatherband w=1 will never occur, the effectiveness for any sortie in this weatherband is irrelevant. It is assumed that any weapon which is feasible for a certain sortie - target combination can be used in the weatherband determined by SELECTOR or in any better weather (higher weatherband).

HEAVY ATTACK uses for each weatherband only the top weapon on Preferred Weapon List. This means that the model will allocate the most cost-efficient weapon feasible in each weatherband. Therefore the data set $E_{i,j,r,w}$ can be reduced by the subscript r such that:

$E^*_{i,j,w}$ = the effectiveness of the most cost-efficient tactic in weatherband w.

Table 5. **EFFECTIVENESS OF THE MOST COST - EFFICIENT TACTIC:** In each weatherband w the first effectiveness value in Table 1 greater than zero is selected.

	w=1	w=2	w=3	w=4	w=5	w=6
$E^*_{i,j,w}$	0	1.580	0.664	0.137	0.137	0.137

Applying the same reasoning on the data set $B_{i,j,r,w}$ and $K_{i,j,r,w}$ yields :

$B^*_{i,j,w}$ = number of weapons used by the most cost-efficient tactic in weatherband w,

$K^*_{i,j,w}$ = type of weapon used by the most cost-efficient tactic in weatherband w.

Table 6. WEAPON LOAD OF THE MOST COST - EFFICIENT TACTIC: In each weatherband the first weapon load value in Table 2 greater than zero is selected.

	w = 1	w = 2	w = 3	w = 4	w = 5	w = 6
$B^{*}_{i,j,w}$	0	2	6	4	4	4

Table 7. WEAPON TYPE OF THE MOST COST - EFFICIENT TACTIC: In each weatherband w the first weapon type in Table 3 not equal to zero is selected.

	w = 1	w = 2	w = 3	w = 4	w = 5	w = 6
$K^{*}_{i,j,w}$	0	17	1	3	3	3

Since each weatherband will occur with the probability PR_w, the averaged effectivness must be

$$\overline{E}_{i,j} = \sum_{w} PR_w \times E^{*}_{i,j,w} = 0.240$$

In general the process of obtaining $\overline{E}_{i,j}$ is a little more complicated than described above because HEAVY ATTACK is permitted to use tactics lower than first order when first order weapon types have been exhausted. This can happen because HEAVY ATTACK is actually a model of protracted war. First order tactics are preferred because they represent the most cost-effective tactic. The war may last for several periods (4 in this Thesis), and it is possible that certain tactics may not be feasible in later periods on account of weapon exhaustion. Suppose for example, that weapon type 3 has been exhausted in a previous time period and is therefore no longer available. The top weapon for weatherband w=4, 5 or 6 is now weapon type 1. The new effectiveness values $E_{i,j,r,w}$ are:

Table 8. **$E_{I,J,R,W}$ - VALUES AFTER WEAPON K=3 IS EXHAUSTED:** Number of targets of type j killed by one sortie of type i using tactic r in weatherband w that is applicable.

i	j	r	w=1	w=2	w=3	w=4	w=5	w=6
1	29	1	NA	NA	NA	NA	NA	NA
1	29	2	0	0	0.664	0.664	0.664	0.664
1	29	3	0	1.580	1.580	1.580	1.580	1.580
1	29	4	0	0	0	0	1.600	1.600

Using the most cost-efficient tactic in each weatherband w gives the following effectiveness values $E^*_{i,j,w}$:

Table 9. **EFFECTIVENESS OF THE NEXT FEASIBLE COST - EFFICIENT TACTIC:** In each weatherband w the first applicable effectiveness value in Table 8 greater than zero is selected.

	w=1	w=2	w=3	w=4	w=5	w=6
$E^*_{i,j,w}$	0.000	1.580	0.664	0.664	0.664	0.664

which results in the averaged effectiveness:

$$\bar{E}_{i,j} = \sum_{w} PR_w \times E^*_{i,j,w} = 0.682 .$$

Note that the effectiveness has increased on account of the lack of weapon type k=3! The SELECTOR output is ordered according to cost-effectivness (not effectiveness), so it is quite possible that tactics far down in the Preferred Weapon List may actually be quite effective. These tactics typically have high associated attrition, but attrition is not considered in HEAVY ATTACK once SELECTOR has done its job.

By considering the same logic, it can be observed that the fourth order tactic on the Preferred Weapon List with $E_{i,j,r,w}$ = 1.600 will never be used. This is because the third order tactic uses the same weapon (in this case weapon type k = 17) in at least the same worst weatherband as tactic r = 4.

D. TIME IN HEAVY ATTACK

Once the effectiveness values $\overline{E}_{i,j}$ are evaluated, the required input data is available in order to optimize the number of sorties assigned to the different target types. For most cases all targets are not killed when the optimization is finished because of the constrained number of sorties in the RAND - model. As in a real war scenario, the outcome of a given attack will influence subsequent target consideration and planning. Only the targets that survived the previous attack will be reconsidered. Weapons are not resupplied and therefore may become exhausted. The current version of HEAVY ATTACK may actually allocate *more* weapons in a given period than are available at the beginning of the period. This is because there is no explicit constraint on weapon usage. The deletion is currently done after each period by computing weapon usage after the optimization for the period is finished. However, a weapon will be deleted in the next period if it is exhausted at the end of the current period.

There is no resupply of targets between periods in HEAVY ATTACK, although there is a facility for reconstituting targets that have already been killed. This will be discussed later. Aircraft are also not resupplied or even directly represented in HEAVY ATTACK: the number of sorties available during each period is a direct input. Each time period represents an attack which changes the input for the following time period.

The fact that the importance of a target will change with time is represented in HEAVY ATTACK by the option of changing the military worth for each target type at the beginning of a new time period. Even though the military worth of a target is known in all future periods, the current sequential time optimization only "sees" the worth of a target for the current time period. Following from this "myopic" way of maximizing the military worth of killed targets it may happen that sorties are assigned in a time period to a target type when its military worth is relatively low. A "global" (or overall) time optimization can be expected to achieve a higher military worth of killed targets. This is discussed later.

E. THE NONLINEAR MODEL IN HEAVY ATTACK

The basic structure of the current model in HEAVY ATTACK for one time period is given by:

Parameter

T_j	number of type j targets available at the beginning of a time period
D_j	number of dead type j targets at the beginning of a time period
V_j	military worth of type j target during the current time period
c_j	target - parameter for type j target
S_i	number of type i sorties available for the current time period
$PROP_i$	proportion of S_i that can be assigned

Variables

$SX_{i,j}$	number of type i sorties that are assigned to type j targets
$KILL_j$	number of type j targets killed in the current time period

Model

$$\text{Max} \quad z = \sum_j V_j \times KILL_j$$

s.t.

$$KILL_j = f\left(T_j, c_j, D_j, \sum_i \overline{E}_{i,j} \times SX_{i,j}\right) \qquad \forall\ j$$

where:

$$f\left(T_j, c_j, D_j, \sum_i \overline{E}_{i,j} \times SX_{i,j}\right) = \left(\frac{T_j}{c_j} - D_j\right) \times \left(1 - e^{-\frac{c_j}{T_j} \times \sum_i \overline{E}_{i,j} \times SX_{i,j}}\right)$$

The above function is the same function as used by RAND [Ref. 2].

$$\sum_j SX_{i,j} \leq PROP_i \times S_i \qquad \forall\ i$$

$$0 \leq KILL_j \leq T_j - D_j \qquad \forall\ j$$

$$0 \leq SX_{i,j} \qquad \forall\ i, j$$

The nonlinear function $f(T_j, c_j, D_j, \sum_i \overline{E}_{i,j} \times SX_{i,j})$ is of the same form as in the RAND - model. The number of targets of type j that are killed and the number of sorties of type i are constrained. The consumption of weapons is not considered in the model itself. After the optimal numbers of sorties are determined by the optimization, the consumption of the different weapon types is evaluated by:

$$\{\text{ consumption of weapon }\}_k = \sum_i \sum_j SX_{i,j} \times \left(\sum_w PR_w \times B^{*}_{i,j,w} \right)$$

where the sum is over all { i, j, w } such that $k = K^{*}_{i,j,w}$.

F. TARGET RECONSTITUTION IN HEAVY ATTACK

The ability to reconstitute killed targets is a common fact in a modern war. HEAVY ATTACK records the number and type of targets as well as the time period when they are destroyed. After each optimization, it determines if targets can be reconstituted and evaluates the maximal number that are possible. A major task in this Thesis has been to determine the conditions under which reconstitution is allowed to happen by analyzing the responsible part of the HEAVY ATTACK source code. HEAVY ATTACK's logic seems to be as outlined below:

Definition of index

j	target type index	$\forall\ j$
p, pp	time period index	$\forall\ p, pp \in \{1, 2, \ldots, n\}$

Parameter

$TIME_p$	length of time period p in days	$\forall\ p$
$RECON_j$	minimum number of days a target has to stay dead	$\forall\ j$
QTY_j	maximum number of targets j that can be reconstituted in 30 days	$\forall\ j$

Aggregated parameter

$PERUP_{j,p}$	index of the last time period considered for reconstitution.

If a target of type j is killed in time period $PERUP_{j,p}$ or earlier, then there is sufficient time available to reconstitute the target so that it once again will be available in period p+1. The parameters $TIME_p$ and $RECON_j$ determine $PERUP_j$ according to the following formula in HEAVY ATTACK:

Let

$$k_{j,\bar{p},p} = \begin{cases} 1 & \text{if } \mathrm{RECON}_j < \sum_{p'=\bar{p}}^{p+1} TIME_{p'} - CEIL\,(0.5 \times TIME_{\bar{p}}) \\ 0 & \text{otherwise} \end{cases} \qquad \forall\ j,\ \bar{p} \le p < n$$

where the function $CEIL$ rounds a real number to the next higher integer value.

$k_{j,\bar{p},p}$ indicates whether targets killed in period $\bar{p}$ are eligible for reconstitution in period p and therefore:

$$PERUP_{j,p} = \sum_{\bar{p}=1}^{p} k_{j,\bar{p},p} \qquad \forall\ j,\ p < n$$

Note that always $PERUP_{j,p} \le p$.

Variables

$KILL_{j,p}$	number of targets type j killed in time period p	$\forall\ j,\ p$
$REBUILD_{j,p}$	maximum number of targets of type j that are reconstituted as live targets in time period p+1	$\forall\ j,\ p < n$

Conditions for Reconstitution

A killed target of type j can be reconstituted if the following 4 conditions are true:

1. at least a fraction of target j was destroyed in the previous or the current time period p,
2. it has been dead for more than some defined time,

3. the total number of targets being reconstituted has to be less than the total number of targets which exceeds the minimum dead time

$$\sum_{p'=1}^{p} REBUILD_{j,p'} \leq \sum_{p'=1}^{PERUP_{j,p}} KILL_{j,p'} \qquad \forall\ j,\ p < n$$

4. the maximum number of targets type j which can be reconstituted at the end of each time period p is given by:

$$REBUILD_{j,p} \leq \frac{QTY_j}{30} \times TIME_{p+1} \qquad \forall\ j,\ p < n$$

where $\frac{QTY_j}{30}$ represents the reconstitution rate per day.

This leads to the following submodel:

$$\max \quad z = \sum_{j} \sum_{p} REBUILD_{j,p}$$

s.t.

$$\sum_{p'=1}^{p} REBUILD_{j,p'} \leq \sum_{p'=1}^{PERUP_{j,p}} KILL_{j,p'} \qquad \forall\ j,\ p < n \qquad \text{(A)}$$

$$REBUILD_{j,p} \leq \frac{QTY_j}{30} \times TIME_{p+1} \qquad \forall\ j,\ p < n \qquad \text{(B)}$$

The interpretation of (A) is that the number of targets of type j rebuilt in period p or before cannot exceed the total number of targets that are killed during or before period $PERUP_{j,p}$. The interpretation of (B) is that the number of targets of type j rebuilt in period p cannot exceed a certain quantity depending on the length of period p and on the target type. There are no targets reconstituted in the last time period p = n .

III. BOUNDS ON WEAPON CONSUMPTION

A. INTRODUCTION OF A WEAPON CONSTRAINT

A desired improvement for the current HEAVY ATTACK model is to add an additional constraint on the utilization of weapons inside the RAND - model.

Two important facts should be recalled:

1. For each sortie - target combination { i, j } and each weatherband there is at most one weapon which can be used.
2. Averaging over all weatherbands is related to the probability that weatherband w might occur at the time sortie type i is assigned to target type j.

Let the upper bound on weapon consumption be defined as:

WP_k total number of weapons of type k available

The required constraint for the consumption on weapons is then:

$$\sum_i \sum_j SX_{i,j} \times \left(\sum_w PR_w \times B^*_{i,j,w} \right) \le WP_k \qquad \forall\ k$$

where the sum is over all { i, j, w } such that $k = K^*_{i,j,w}$

B. REVISED MODEL OF HEAVY ATTACK

Reconstitution can be included in the RAND - model. Instead of considering reconstitution as a computational "bookkeeping" process, it can be part of the optimization. To accomplish this, it is necessary to define a new variable for the number of dead targets such that the time period as an additional dimension is represented by a second subscript:

$D_{j,p}$ is the total number of targets of type j killed in time periods < p *less* the number of targets that are reconstituted during this time :

$$D_{j,p} = \sum_{p'=1}^{p-1} (KILL_{j,p'} - REBUILD_{j,\,p'}) \qquad \forall\ j,\ p$$

The military worth of a target is also time dependent:

$V_{j,\,p}$ military worth of a target type j in time period p

Embellished Thesis Model (solved sequentially for p = 1, 2, 3,..., n)

$$\text{Max} \quad z_p = \sum_j (V_{j,p} \times KILL_{j,p})$$

s.t.

$$KILL_{j,p} = f\left\{T_j\,,\ c_j\,,\ D_{j,p}\,,\ \sum_i SX_{i,j} \times \left(\sum_w PR_w \times E^*_{i,j,w}\right)\right\} \qquad \forall\ j$$

where : $f\{\ldots\}$ is one of three functions discussed in the next chapter.

$$KILL_{j,p} \le T_j - D_{j,p} \qquad \forall\ j$$

$$D_{j,p} = \sum_{p'=1}^{p-1} (KILL_{j,p'} - REBUILD_{j,p'}) \qquad \forall\ j$$

$$\sum_{p'=1}^{p} REBUILD_{j,p'} \le \sum_{p'=1}^{PERUP_{j,p}} KILL_{j,p'} \qquad \forall\ j$$

$$\sum_j SX_{i,j} \le PROP_i \times S_i \qquad \forall\ i$$

$$\sum_i \sum_j \left\{SX_{i,j} \times \left(\sum_w PR_w \times B^*_{i,j,w}\right)\right\} \le WP_k \qquad \forall\ k$$

where the sum is over all $\{\, i,\ j,\ w \,\}$ such that $k = K^*_{i,j,w}$

$$0 \leq SX_{i,j} \qquad \forall\ i, j$$

$$0 \leq KILL_{j,p} \qquad \forall\ j$$

$$0 \leq D_{j,p} \qquad \forall\ j$$

$$0 \leq REBUILD_{j,p} \qquad \forall\ j$$

where the upper bound on $REBUILD_{j,p}$ is such that:

$$REBUILD_{j,p} \begin{cases} \leq \dfrac{QTY_j}{30} \times TIME_{p+1} & \text{if } p < n \qquad \forall\ j \\ = 0 & \text{if } p = n \qquad \forall\ j \end{cases}$$

The model was written in the *General Algebraic Modeling System (GAMS)* [Ref. 3]. All optimization problems throughout the Thesis are solved with the nonlinear programing solver *MINOS - Version 5.0* [Ref. 4]. A database for 2 sortie-, 26 target- and 29 weapon-types was provided [Ref. 5] in order to compare the results by using three different objective functions, each over four time periods.

IV. LINEAR VERSUS NONLINEAR MODEL

In this chapter the derivation of the nonlinear objective function used by RAND is given. In addition two alternatives are represented by introducing the Washburn-Equation and the linear case in which the number of killed targets is proportional to the number of assigned sorties. Each of the three objective functions is used in the model described in the previous chapter for sequentialy optimizing sortie assignments over four time periods. In order to compare the effect of the three objective functions, a measurement for the diversity of the allocated kill capability is defined.

A. RAND EQUATION

If K_j represents the total number of killed targets of type j then the objective function used in the RAND - model can be derived from the differential equation:

$$\frac{\mathrm{d}\,K_j}{\mathrm{d}\,X_j} = 1 - c_j \times \frac{K_j}{T_j} \tag{A}$$

$$\text{where} \quad X_j = \sum_i \overline{E}_{i,j} \times SX_{i,j} \quad \text{and} \quad 0 \leq c_j \leq 1$$

The differential equation (A) with the initial condition $K_j(X_j = 0) = D_j$ has the solution:

$$K_j = \frac{T_j}{c_j} \times \left\{ 1 - (1 - c_j \times \frac{D_j}{T_j}) \times e^{-\frac{c_j}{T_j} \times X_j} \right\}$$

Instead of bounding K_j by

$$D_j \leq K_j \leq T_j$$

let $KILL_j$ be the number of targets killed in excess of D_j :

$$KILL_j = K_j - D_j$$

so that

$$0 \leq KILL_j \leq T_j - D_j$$

which leads to the final result:

$$KILL_j = \left(\frac{T_j}{c_j} - D_j \right) \times \left(1 - e^{- \frac{c_j}{T_j} \times X_j} \right)$$

B. LINEAR EQUATION

A special case for the differential equation (A) appears when $c_j = 0$:

then

$$\frac{d\,K_j}{d\,X_j} = 1$$

which yields:

$$K_j = X_j + D_j$$

so that

$$D_j \leq K_j \leq T_j$$

or by using

$$KILL_j = K_j - D_j$$

so that

$$0 \leq KILL_j \leq T_j - D_j$$

where the final solution represents the linear case:

$$KILL_j = X_j$$

Figure 1 illustrates the influence of the target parameter c_j on the function $KILL_j = f(X_j)$.

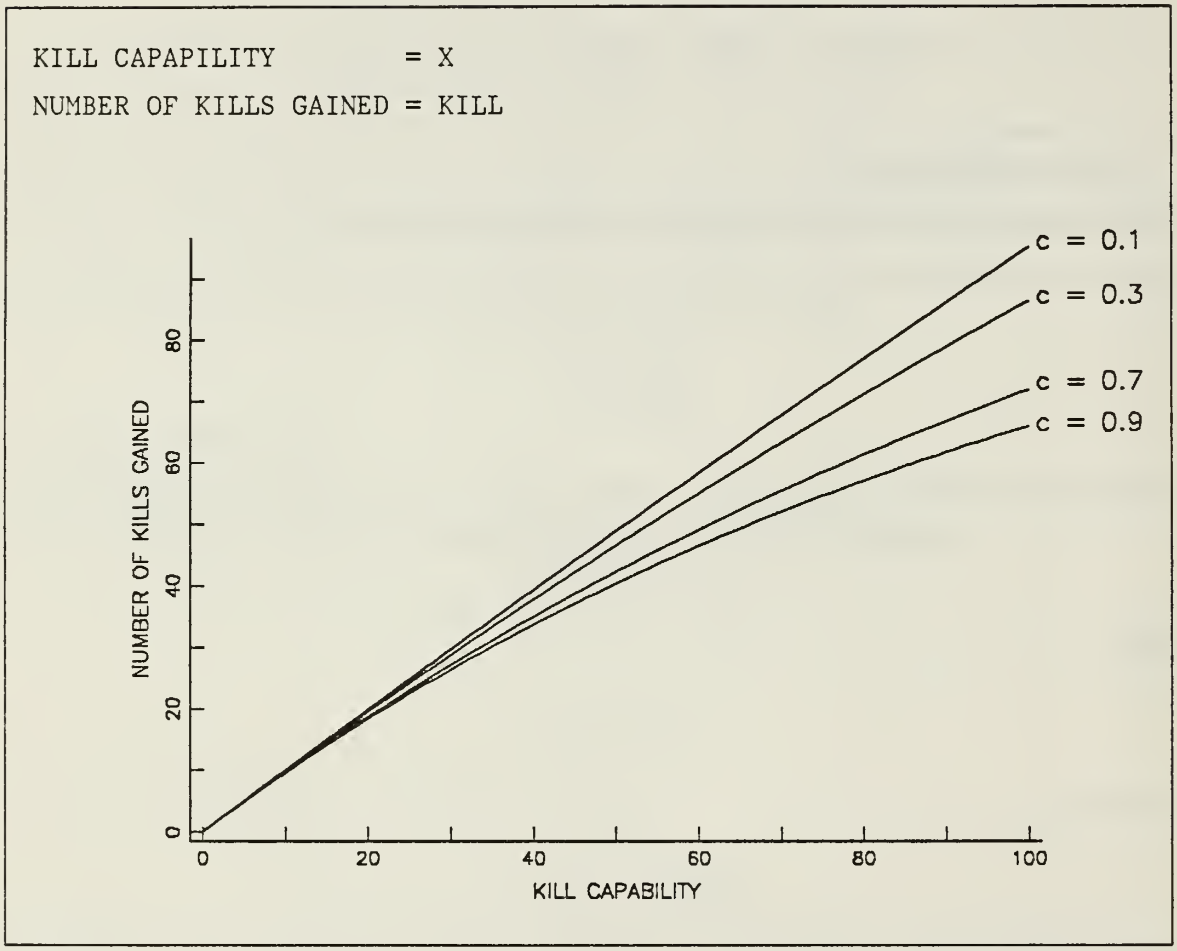

Figure 1. **Influence of the target parameter c on the RAND-Equation:** The solution of the differential equation used in the RAND-model is graphically shown for 4 different target parameters c.

The parameter c_j has no direct physical motivation. The model considered in the next section also contains a single parameter, but the parameter can be motivated physically.

C. WASHBURN EQUATION

The Washburn - Equation [Ref. 6: p. 25] defines the differential $\frac{\mathrm{d}\,K_j}{\mathrm{d}\,X_j}$ in the following way:

$$\frac{\mathrm{d}\,K_j}{\mathrm{d}\,X_j} = \text{Probability } \{\text{ attacking a live target }\}$$

or equivalently:

$$\frac{\mathrm{d}\,K_j}{\mathrm{d}\,X_j} = \frac{\{\text{ number of live targets }\}}{\{\text{ number of targets that look alive }\}}$$

This leads to the differential equation:

$$\frac{\mathrm{d}\,K_j}{\mathrm{d}\,X_j} = \frac{T_j - K_j}{T_j - K_j + \alpha_j \times K_j} \tag{B}$$

where α_j is a constant proportion of killed targets, which have the property to appear live to a potential attacker.

The differential equation (B) with the initial condition $K_j(X_j = 0) = D_j$ has the solution:

$$K_j = T_j \times \left\{ 1 - \left(1 - \frac{D_j}{T_j} \right) \times e^{\frac{(1-\alpha_j)\times(K_j - D_j) - X_j}{\alpha_j \times T_j}} \right\}.$$

Using $KILL_j$ instead of K_j such that:

$$KILL_j = K_j - D_j$$

leads to the implicit solution for the Washburn - Equation as:

$$KILL_j = (\,T_j - D_j\,) \times \left(1 - e^{\frac{(1-\alpha_j)\times KILL_j - X_j}{\alpha_j \times T_j}} \right).$$

The difference between the two differential equations (A) and (B) for two different target parameters is shown in Figure 2 on page 24 . Observe that for target parameter c close to 0 or 1 the Washburn-equation tends to behave similarly to the RAND-equation.

Target parameter α is denoted in the figure by c.

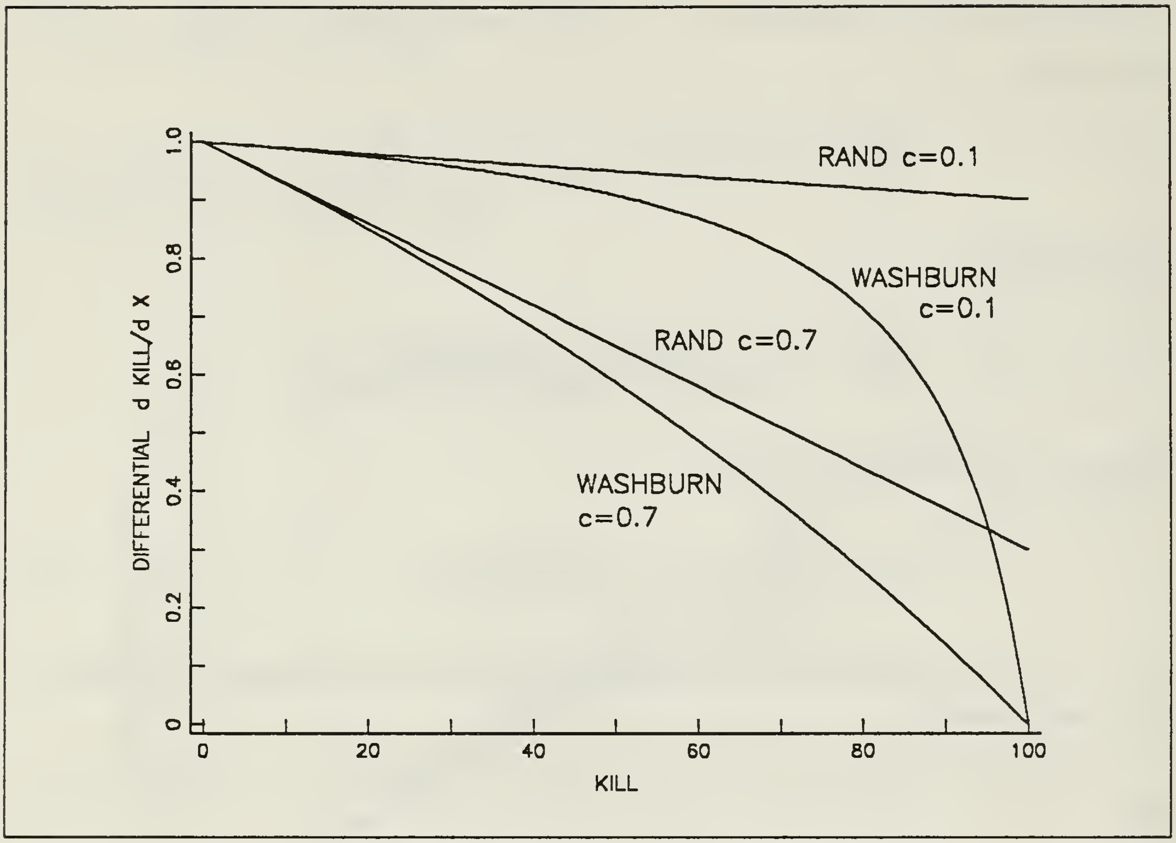

Figure 2. RAND- and Washburn-Diff. Equation with varied parameter c: The two differential equations are shown for 2 different target parameters c. Because the solution of the Washburn-Equation can be given only in an implicit form, the differential equations are shown rather than their solutions.

The influence of the three different objective functions on the RAND-model using the same input data is shown in Figure 3.

The total worth of killed targets decreases with time for each objective function. The main reason for this is that in the first time period sorties are assigned to those target types for which the effectiveness is highest. When all targets are killed, sorties are then assigned in the following time periods to the remaining targets for which the effectiveness is less. As a result, more and more sorties need to be allocated in order to gain the same number of killed targets. The number of reconstituted targets available at the beginning of the second or third period is relatively small or even zero and can therefore be neglected at this point. Since the variation in the number of sorties and in the magnitude of the target values is too small to compensate for this effect, a declining trend in the objective function value over time for all three cases is observed.

Note that the Washburn-Equation always yields a smaller value than the RAND-Equation. This follows from the fact that the Washburn-Equation declines faster than the RAND-Equation for the same target parameter c as shown in Figure 2. The linear equation is larger than either one. The most important difference is not in the absolute level of target value killed, but rather in the influence of the objective function on the distribution of sorties over targets. This subject is taken up in the next section.

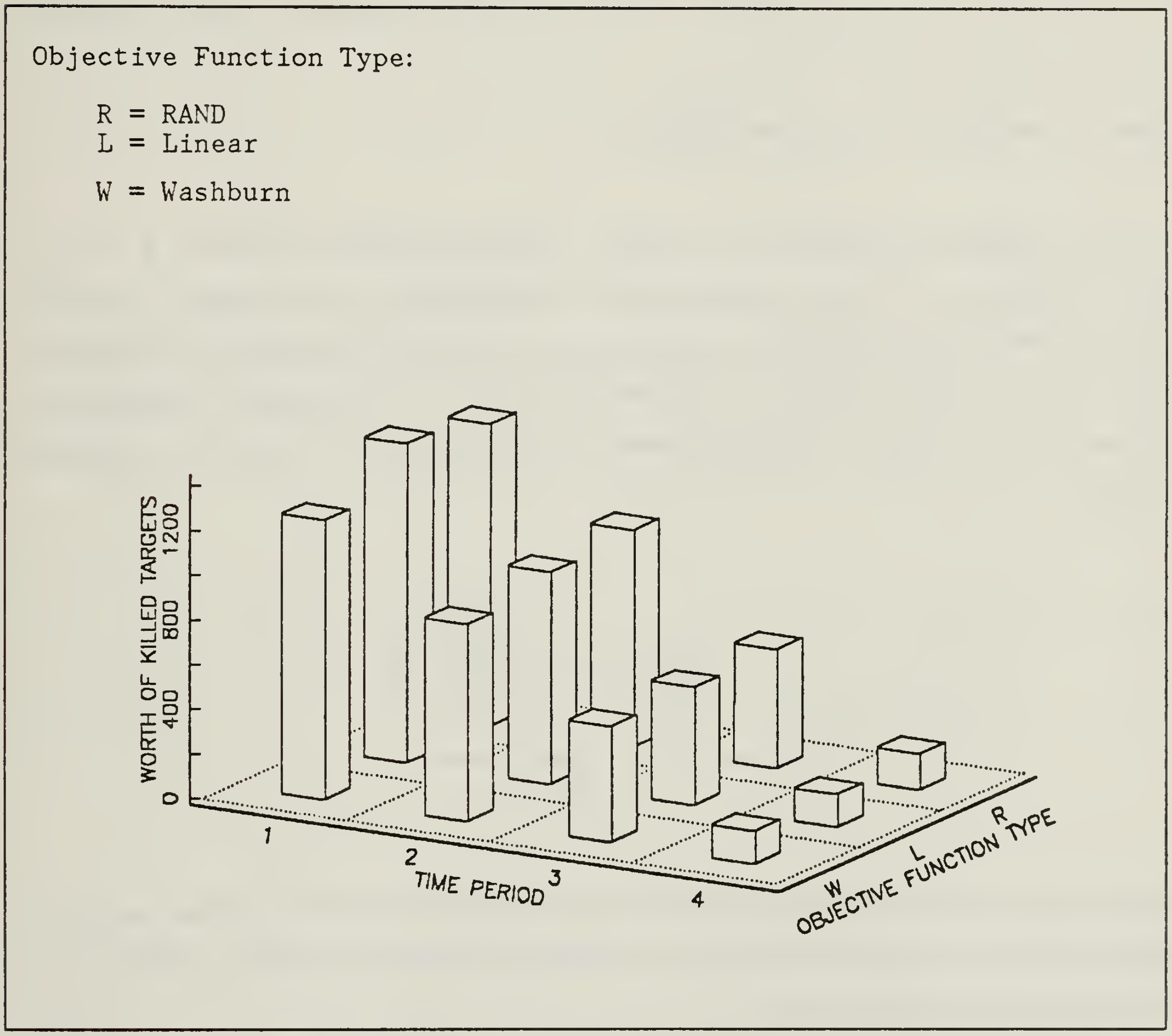

Figure 3. Total Military Worth of Killed Targets: represented for each different objective function and each time period by the height of the respective block in the figure.

D. DIVERSITY OF KILLED TARGETS

An important reason for USAF to use a nonlinear objective function is to avoid an undesired concentration of attacking sorties on a few targets. In analysing the effect of the three different objective functions on the optimization, a measurement is needed in order to indicate how many of the allocated sorties are spread over different targets.

In information theory the function

$$h(\mathbf{p}) = \sum_i \left(p_i \times \log \frac{1}{p_i} \right)$$

where $\mathbf{p} = (p_1, p_2, \dots, p_n)$ and $\sum_i p_i = 1$

is used to express the diversity or "entropy" of the probability distribution $\mathbf{p} = \{ p_i \}$. Observe that $h(\mathbf{p}) = 0$ when $\mathbf{p}$ concentrates all probability in one element. The maximum possible value when $\mathbf{p}$ has n elements occurs when they are all equal, in which case $h(\mathbf{p}) = \log n$. The diversity $h(\mathbf{x})$ of an arbitary set $\{ x_j \}$ of nonnegative members can be measured by simply normalizing them so that they sum to 1 and then computing entropy:

$$h(\mathbf{x}) = \frac{\sum_j X_j \times \log \left[\dfrac{\sum_j X_j}{X_j} \right]}{\sum_j X_j}$$

The diversity of values $h(\mathbf{x})$ gained from the same input data and model as used in the previous chapter is depicted in Figure 4. Since the number of targets n equals 26, the maximum diversity value will be

$$h(\mathbf{x})_{\max} = 3.26$$

Figure 4 makes it clear that the Linear objective function has a lower diversity value than the other two. This is to be expected, and in fact one of the main reasons for using a nonlinear objective in the first place was to avoid low diversity values. However, note that:

1. The Linear diversity is not 0; that is, several target types are still attacked.
2. None of the objective functions achieves complete (3.26) diversity.

The differences emerge most strongly in period 3. Only 4 target types are attacked when the linear model is used, or 6 with the RAND-model. 16 different target types are attacked when the Washburn-equation is used; this is in keeping with the idea that the Washburn-equation is the most "non-linear" of the three (see Figure 2). The three models differ much less in period 1,2 or 4.

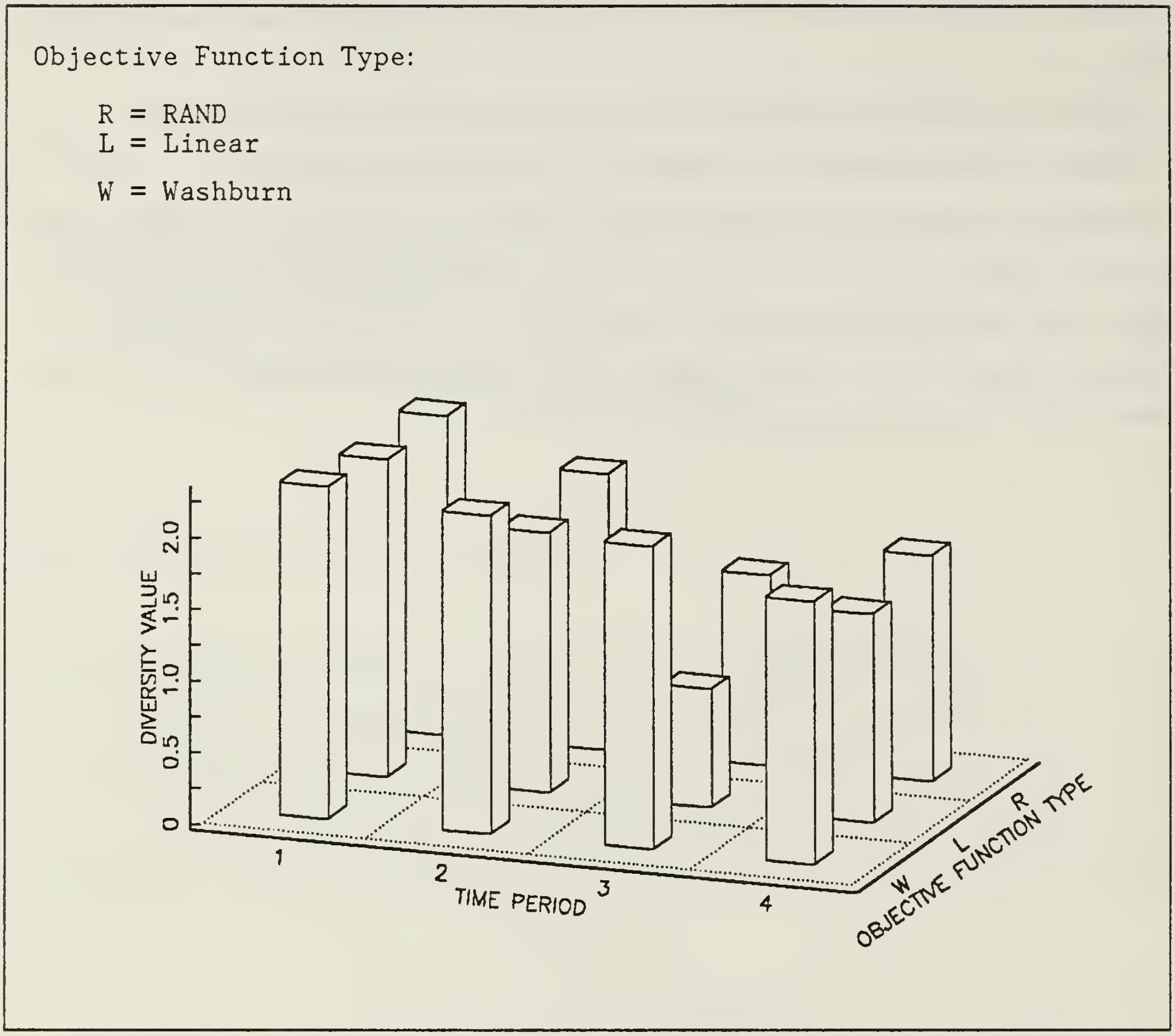

Figure 4. **Diversity of killed targets for different objective functions:** The height of each block illustrates to how many different target types (out of 26) sorties are allocated at different time periods by using each of the three objective functions.

V. ALLOCATION OF SECONDARY WEAPONS

A. COST-EFFICIENCY VERSUS KILL-EFFECTIVENESS

Cost considerations are finished once SELECTOR has established the Preferred Weapon List. Although this list contains different tactics, ordered in terms of cost-efficiency, HEAVY ATTACK only uses the top one on the list which is feasible. The only time at which HEAVY ATTACK may proceed to a succeeding tactic appears, as mentioned before, when a weapon has been exhausted in earlier periods.

As a second revision of HEAVY ATTACK, the model is changed to continue target attacks after the weapon type used by the most cost-effective tactic has been exhausted, using those weapons still on hand.

B. A NONCONVEX CONSTRAINT

The model discussed in the previous chapter requires that only the tactic on the top of SELECTOR's Preferred Weapon List can be used. Once the corresponding weapon type is depleted further attacks by that sortie type in that weatherband against that target type are impossible. The idea in this section is to relax this strict requirement to permit using whatever tactic is highest on SELECTOR's list *among those whose weapons have not been exhausted.*

Implementing this logic in the existing model requires a modification of the variable $SX_{i,j}$:

$SX_{i,j,r,w}$ = number of sorties of type i assigned to target of type j which use tactic r in weatherband w

The probability that all sorties of type i assigned to target of type j will attack the target in weatherband w has to be equal to the probability that weatherband w occurs at that time:

$$\sum_{r} SX_{i,j,r,w} = PR_{w} \times \sum_{r} \sum_{w'} SX_{i,j,r,w'}$$

Upon these redefined variables for the number of assigned sorties, it is possible to determine the utilization of each weapon type:

let $WEAP_k$ be the consumption of all weapons of type k

$$\text{then} \qquad WEAP_k = \sum_i \sum_j \sum_r \sum_w (B_{i,j,r,w} \times SX_{i,j,r,w}) \qquad \forall\ k$$

where the sum is over all $\{\ i,\ j,\ r,\ w\ \}$ such that $k\ =\ K_{i,j,r,w}$.

In order to assign sorties using less cost-effective tactics, $SX_{i,j,r,w}$ must be 0 unless the weapon types corresponding to all more cost-effective tactics are exhausted. The following constraint will enforce this logic:

$$0\ =\ SX_{i,j,r,w} \times \sum_{r'=1}^{r-1} (WP_k\ -\ WEAP_k) \qquad \forall\ i,\ j,\ r,\ w \quad \text{(C)}$$

where $k\ =\ K_{i,j,\,r',\,w}$.

The above constraint requires that at least one of the two factors on the right hand side of the equation equals zero, so either no sorties are assigned (first factor zero) or else all more cost-effective weapons are exhausted (second factor zero). The constraint thus enforces the desired logic, but there is a disadvantage in using it. The disadvantage is that the function on the right hand side of (C) is not only nonlinear (products of variables are involved) but nonconvex. Without constraint convexity, there is no guarantee that the locally optimal solutions achieved by the *MINOS* solver are globally optimal. There is some evidence, however, that globally optimal solutions are actually being attained. For one thing, employing constraint (C) always results in a higher objective function value than when only the most cost-efficient tactic is permitted. In addition, some experiments were performed where the improved model was changed into a linear model by linearizing the objective function at the optimal solution. The nonconvex constraint was then converted into a linear constraint by using integer variables. The optimal solution of this linearized model was identical to the solution gained by the nonlinear model with the nonconvex constraint.

C. REVISED MODEL

The mathematical model is solved sequentially for p = 1, 2,..., n.

$$\text{Max} \quad z_p = \sum_j (V_{j,p} \times KILL_{j,p})$$

s.t.

$$KILL_{j,p} = \left(\frac{T_j}{c_j} - D_{j,p} \right) \times \left(1 - e^{-\frac{c_j}{T_j} \times X_j} \right) \qquad \forall\ j$$

$$\text{where} \quad X_j = \sum_i \sum_r \sum_w (E_{i,j,r,w} \times SX_{i,j,r,w})$$

$$KILL_{j,p} \leq T_j - D_{j,p} \qquad \forall\ j$$

$$D_{j,p} = \sum_{p'=1}^{p-1} (KILL_{j,p'} - REBUILD_{j,p'}) \qquad \forall\ j$$

$$\sum_{p'=1}^{p} REBUILD_{j,p'} \leq \sum_{p'=1}^{PERUP_{j,p}} KILL_{j,p'} \qquad \forall\ j$$

$$\sum_j \sum_r \sum_w SX_{i,j,r,w} \leq PROP_i \times S_i \qquad \forall\ i$$

$$WEAP_k = \sum_i \sum_j \sum_r \sum_w (B_{i,j,r,w} \times SX_{i,j,r,w}) \qquad \forall\ k$$

where the sum is over all $\{ i,\ j,\ r,\ w \}$ such that $k = K_{i,j,r,w}$

$$0 = SX_{i,j,r,w} \times \sum_{r'=1}^{r-1} (WP_k - WEAP_k) \qquad \forall\ i,\ j,\ r,\ w$$

where $k = K_{i,j,r',w}$

$$\sum_{r} SX_{i,j,r,w} = PR_w \times \sum_{r} \sum_{w'} SX_{i,j,r,w'} \qquad \forall\ i,\ j,\ w$$

$$0 \leq SX_{i,j,r,w} \qquad \forall\ i,\ j,\ r,\ w$$

$$0 \leq KILL_{j,p} \qquad \forall\ j$$

$$0 \leq D_{j,p} \qquad \forall\ j$$

$$0 \leq REBUILD_{j,p} \qquad \forall\ j$$

where the upper bound on $REBUILD_{j,p}$ is such that:

$$REBUILD_{j,p} \begin{cases} \leq \dfrac{QTY_j}{30} \times TIME_{p+1} & \text{if } p < n \qquad \forall\ j \\ = 0 & \text{if } p = n \qquad \forall\ j \end{cases}$$

$$0 \leq WEAP_k \leq WP_k \qquad \forall\ k$$

The introduced relaxation will be used in the further revision of HEAVY ATTACK considered in the next chapter.

VI. GLOBAL VERSUS MYOPIC TIME OPTIMIZATION

A. TIME-DEPENDENT MILITARY WORTH OF TARGETS

When HEAVY ATTACK optimizes the allocation of sorties for each time period, it doesn't take advantage of the fact that the military worth of each target and each time period is known prior to running the optimization. The decision, which target type should be given a high priority to attack, is based on a comparison of military values of different target types restricted to the current time period. Although military worth of a target is given as a function of time, HEAVY ATTACK doesn't recognize the most favorable time for attacking a certain target type. This "myopic view" is caused by restricting the optimization to the time interval covered by one period.

It seems worthwhile to consider an optimization covering all time periods at once. This "global" optimization is expected to spend resources even more effectively than before, so that the total sum of gained military worth of killed targets might become higher compared to sequential time optimization. In addition, it can be expected that the number and type of killed targets in each time period will change.

The third revision for HEAVY ATTACK as presented in this chapter doesn't require major changes to the previously discussed model. A subscript for time is added to the variable $SX_{i,j,r,w}$:

$SX_{i,j,r,w,p}$	number of sorties of type i assigned to target type j by using tactic type r in weatherband w and in time period p

The resources on sorties available needs to be defined as a function of sortie type and time:

$S_{i,p}$	maximum number of sorties type i available in period p
$PROP_{i,p}$	proportion of $S_{i,p}$ that can be assigned

Computing time increases with the number of time periods covered.

B. GLOBAL MODEL

The mathematical model is shown below. The realization of this model in *GAMS*, including all inputs, is given in the Appendix.

$$\text{Max} \quad z = \sum_j \sum_p (V_{j,p} \times KILL_{j,p})$$

s.t.

$$KILL_{j,p} = \left(\frac{T_j}{c_j} - D_{j,p} \right) \times \left(1 - e^{-\frac{c_j}{T_j} \times X_{j,p}} \right) \qquad \forall \; j, \; p$$

where $X_{j,p} = \sum_i \sum_r \sum_w (E_{i,j,r,w} \times SX_{i,j,r,w,p})$

$$KILL_{j,p} \le T_j - D_{j,p} \qquad \forall \; j, \; p$$

$$D_{j,p} = \sum_{p'=1}^{p-1} (KILL_{j,p'} - REBUILD_{j,p'}) \qquad \forall \; j, \; p$$

$$\sum_{p'=1}^{p} REBUILD_{j,p'} \le \sum_{p'=1}^{PERUP_{j,p}} KILL_{j,p'} \qquad \forall \; j, \; p$$

$$\sum_j \sum_r \sum_w SX_{i,j,r,w,p} \le PROP_{i,p} \times S_{i,p} \qquad \forall \; i, \; p$$

$$WEAP_k = \sum_i \sum_j \sum_r \sum_w \left(B_{i,j,r,w} \times \sum_p SX_{i,j,r,w,p} \right) \qquad \forall \; k$$

where the sum is over all $\{ i, \; j, \; r, \; w \}$ such that $k = K_{i,j,r,w}$

$$0 = SX_{i,j,r,w} \times \sum_{r'=1}^{r-1} (WP_k - WEAP_k) \qquad \forall \; i, \; j, \; r, \; w$$

where $k = K_{i,j,r',w}$

$$\sum_{r} SX_{i,j,r,w,p} = PR_w \times \sum_{r} \sum_{w'} SX_{i,j,r,w',p} \qquad \forall\ i, j, w, p$$

$$0 \leq SX_{i,j,r,w,p} \qquad \forall\ i, j, r, w, p$$

$$0 \leq KILL_{j,p} \qquad \forall\ j, p$$

$$0 \leq D_{j,p} \qquad \forall\ j, p$$

$$0 \leq REBUILD_{j,p} \qquad \forall\ j$$

where the upper bound on $REBUILD_{j,p}$ is such that:

$$REBUILD_{j,p} \begin{cases} \leq \dfrac{QTY_j}{30} \times TIME_{p+1} & \text{if p < n} \qquad \forall\ j \\ = 0 & \text{if p = n} \qquad \forall\ j \end{cases}$$

$$0 \leq WEAP_k \leq WP_k \qquad \forall\ k$$

C. RESULTS AND COMPARISONS

The above model was too large to be run in *GAMS* on available computer equipment at reasonable cost with the same size of input data used previously. Therefore the number of target types were reduced from 26 to 13. Other efforts were also made to decrease required computing time.

Table 10, Table 11 and Figure 5 compare the results of the global and myopic sequential optimizations. The global optimization achieves more target value killed; the percentage gain for the global approach is (1358.0 - 1123.0)/1123.0 = 20.9 %. Comparing the target values of target type 5 and 27 over all 4 periods shows that the highest target value occurs in period 3. The global optimization realizes this fact by destroying all available targets at that time. While both target types, especially target type 5, have a relatively high target value in the first time period, most of these targets are therefore killed by myopic optimization in the first period.

Table 10. NUMBER OF KILLED TARGETS: The table shows the number of killed targets achieved by sequential and global optimization as well as the respective target value for each time period.

	Time Period 1			Time Period 2		
Target Type	Target Value	Killed Targets		Target Value	Killed Targets	
		Myopic	**Global**		**Myopic**	**Global**
TG 5	10	17.3	0.5	14	1.2	1.1
TG 8	10	13.0	13.0	10	0.0	0.0
TG 10	4	0.0	0.0	7	0.0	0.0
TG 11	7	0.0	9.6	9	0.0	0.0
TG 12	7	0.0	0.0	12	0.0	0.0
TG 13	4	0.0	2.2	5	0.0	0.0
TG 14	20	2.0	2.0	15	0.0	0.0
TG 22	2	0.0	0.0	2	0.0	0.0
TG 24	2	0.0	0.0	7	0.1	0.0
TG 25	5	0.0	0.0	12	22.3	26.6
TG 27	4	19.1	0.0	7	1.9	0.0
TG 29	7	0.0	0.0	7	0.0	0.0
TG 34	5	8.6	0.0	5	9.4	0.0
	Time Period 3			Time Period 4		
Target Type	Target Value	Killed Targets		Target Value	Killed Targets	
		Myopic	**Global**		**Myopic**	**Global**
TG 5	18	1.0	18.0	1.0	1.0	2.0
TG 8	10	0.0	0.0	0.7	0.0	0.0
TG 10	10	5.4	0.0	3.1	23.6	26.3
TG 11	10	4.3	0.0	2.1	3.5	0.0
TG 12	18	0.0	0.0	2.1	0.0	0.0
TG 13	7	4.0	1.7	1.0	0.0	0.1
TG 14	10	0.	0.0	0.7	0.0	0.0
TG 22	2	0.0	0.0	2.0	6.0	6.0
TG 24	10	2.2	1.5	2.5	0.4	1.3
TG 25	10	5.5	1.1	0.9	0.0	0.0
TG 27	8	0.0	21.0	2.0	0.0	0.0
TG 29	8	0.0	0.0	1.0	0.0	0.0
TG 34	8	0.0	18.0	0.7	0.0	0.0

Table 11. **MILITARY WORTH OF KILLED TARGETS:** gained by sequential and by global optimization is given for each time period and as a total sum.

	Myopic Optimization	Global Optimization
Time Period 1	462.8	251.3
Time Period 2	345.5	333.8
Time Period 3	220.1	674.0
Time Period 4	94.6	98.9
Total Worth of Killed Targets	**1123.0**	**1358.0**

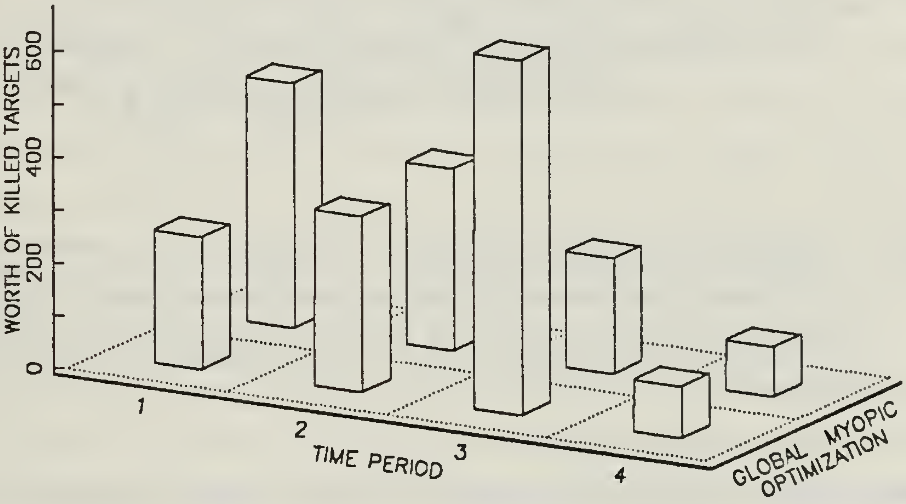

Figure 5. **Distribution of Military Worth of Killed Targets:** The height of each block represents the numerical value given in Table 11 depending on the time period and on the kind of optimization used.

Both the global and the myopic models utilize secondary weapons. Figure 6 shows weapon usage in the global model. Note that weapon type WP7 is used extensively in situations where more cost-effective weapons are exhausted.

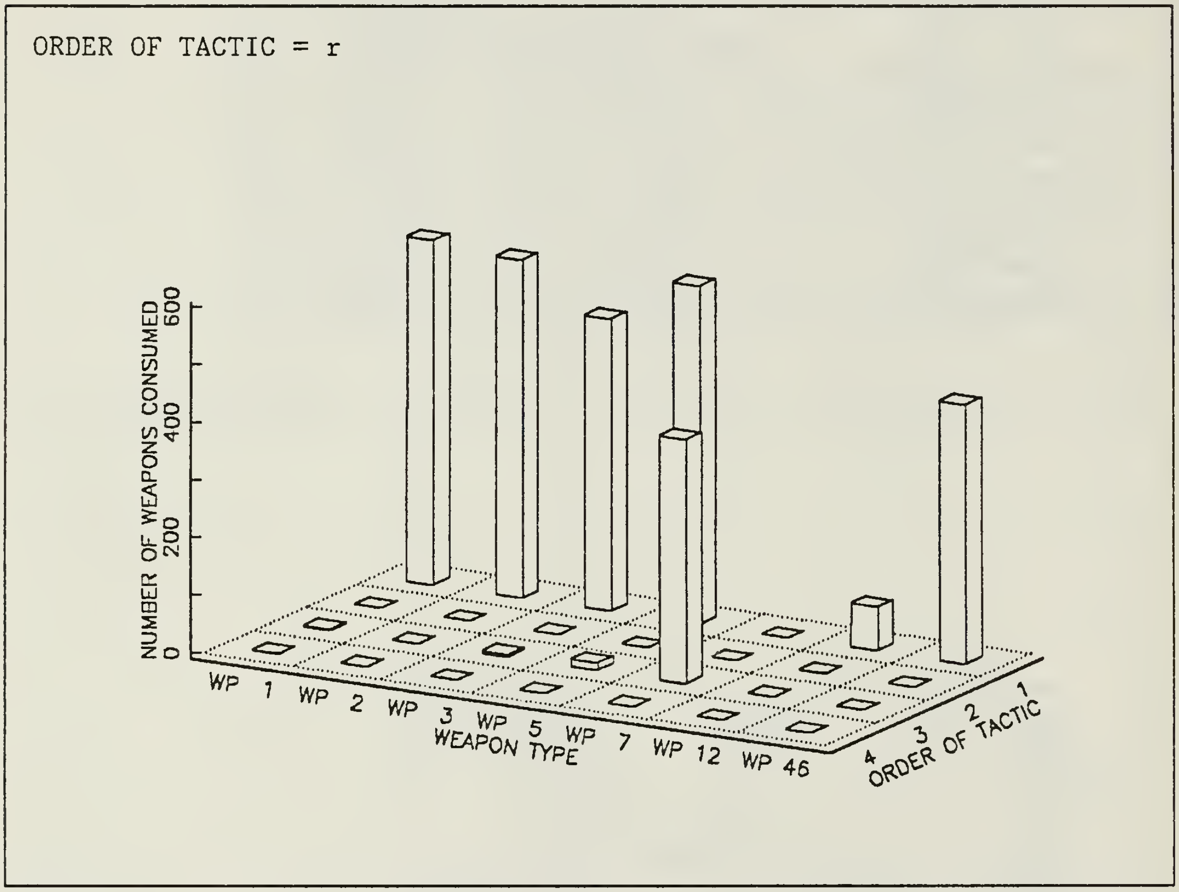

Figure 6. Allocation of Secondary Weapons: The height of each block represents the number of weapons utilized by the global optimization. A significant number of weapon type WP7 is used by tactics of order r = 3. This is only possible when weapons used by tactics of order r = 1 and r = 2 are exhausted.

A more detailed report of the solution is given in the SOLVE SUMMARY of *GAMS* in the Appendix.

VII. CONCLUSIONS

In the first revision of the current HEAVY ATTACK model, a weapon constraint is added and three different objective functions are compared. The objective function best used in the model depends on the priorities of the user:

1. Using a linear objective function instead of a nonlinear one has the advantage of simplicity and consequent computational efficiency. A disadvantage is a less dispersed allocation of sorties to different targets.
2. Using the Washburn - Equation instead of the RAND - Equation has the advantage of using a well defined target parameter. The dispersion of attacked target types might be somewhat less influenced due to changes in the input data.

In the second revision the current philosophy of using the most cost-efficient tactic is relaxed such that less cost-efficient tactics can be utilized within a time period. With this revision, tactics not at the top of the Preferred Weapon List (SELECTOR output) can be utilized if all more cost-effective tactics are infeasible due to weapon exhaustion. This revision is particularly important when there is a small number of time periods, since the same capability already exists *between* time periods.

The third revision replaces sequential optimization (current practice) with global optimization. The comparison between sequential and global optimization by using the same input data shows a qualitative difference in the achieved results. There is a definite indication that sequential time optimization tends to achieve military success in the beginning of the war by sacrificing the potential for later success. Global optimization tends to husband weapons and even targets (in cases where target value increases with time) for later periods in the war. An argument for global optimization can be based on the fact that it is more efficient in killing targets with large military values. On the other hand, it could also be argued that sequential optimization is more likely to imitate what will actually happen, "optimal" or not. In any case, if global optimization is used, then the distribution of the value of destroyed targets seems to be much more time dependent than is recognized by the current method of sequential optimization.

All revisions introduced in this Thesis result in gaining of more military worth. USAF's general objective is to determine their future need of weapons rather than to maximize the military worth of killed targets. With the revisions described above, utilization of weapons plays a more important and direct role in the optimization, especially when more than one tactic is considered. The developed models are intended to provide the necessary structure to embellish HEAVY ATTACK for this purpose.

APPENDIX GLOBAL OPTIMIZATION MODEL

```

*************************************************************************
*                                                                       *
* Math.Model: Klaus Wirths                               February 1989 *
*                                                                       *
* File Name : P H C R    G A M S                                        *
*                                                                       *
*                                                                       *
* Remark    : This Model is an improved version of the HEAVY ATTACK    *
*             model; it contains a subset of a larger database.        *
*                                                                       *
* Specification:   RAND - Equation                                      *
*                  Multi-Weapon Optimization                            *
*                  Multi-Time Period (Global) Optimization              *
*                                                                       *
*                                                                       *
* Reference : Dennis M. Coulter, Maj, USAF                              *
*             War, Mobilization & Munitions Division                    *
*             Directorate of Plans, DCS/P&Q                             *
*                                                                       *
*                                                                       *
*             Sortie Allocation by a Nonlinear Programming Model        *
*             for Determining a Munitions Mix                           *
*             R.J.Clasen, G.W.Graves and J.Y.Lu                         *
*             RAND, Santa Monica                         March 1974     *
*                                                                       *
*************************************************************************

SET
     I     aircraft type index    / AC1 * AC2 /

     J     target type index      / TG5
                                    TG8
                                    TG10
                                    TG11
                                    TG12
                                    TG13
                                    TG14
                                    TG22
                                    TG24
                                    TG25
                                    TG27
                                    TG29
                                    TG34 /

     K     weapon type index      / WP1
                                    WP2
                                    WP3
                                    WP4
                                    WP5
                                    WP6
                                    WP7
                                    WP8
                                    WP9
                                    WP10
                                    WP11
                                    WP12
                                    WP15
                                    WP18
                                    WP19
                                    WP21
                                    WP22
                                    WP24
                                    WP25
                                    WP27
                                    WP34
                                    WP42
                                    WP45
                                    WP46  /

     W     weatherband type index / WB1 * WB6 /

     R     order of preferred weapon type / OD1 * OD4 /

     P     time period index   / PER1 * PER4 /                       ;


ALIAS (J,JP)                                                         ;

ALIAS (R,RP)                                                         ;

ALIAS (P,PP)                                                         ;

ALIAS (P,PPP)                                                        ;

ALIAS (W,WPP)                                                        ;


**  Definition of TARGET Parameters

PARAMETERS

     T(J)  total number of target type J

***  all entries for T(J) has to be nonzero values  ***

                                  / TG5   18
                                    TG8   13
                                    TG10  29
                                    TG11  32
                                    TG12   3
                                    TG13   4
                                    TG14   2
                                    TG22   6
                                    TG24   3
                                    TG25  51
                                    TG27  21
                                    TG29   9
                                    TG34  18  /



     C(J)  TARGET parameter                          0 < C < 1

                                  / TG5    0.2
                                    TG8    0.1
                                    TG10   0.2
                                    TG11   0.1
                                    TG12   0.1
                                    TG13   0.3
                                    TG14   0.1
                                    TG22   0.2
                                    TG24   0.8
                                    TG25   0.3
                                    TG27   0.7
                                    TG29   0.1
                                    TG34   0.2   /                    ;


 TABLE   V(J,P)  value of target type J

                    PER1  PER2  PER3  PER4
           TG5       10    14    18    1.0
           TG8       10    10    10    0.7
           TG10       4     7    10    3.1
           TG11       7     9    10    2.1
           TG12       7    12    18    2.1
           TG13       4     5     7    1.0
           TG14      20    15    10    0.7
           TG22       2     2     2    2.0
           TG24       2     7    10    2.5
           TG25       5    12    10    0.9
           TG27       4     7     8    2.0
           TG29       7     7     8    1.0
           TG34       5     5     8    0.7



**  Definition of Sortie numbers

   . TABLE  S(I,P)   maximum number of sorties for AC type I

                   PER1   PER2   PER3   PER4
             AC1   180    200    150    300
             AC2   180    200    150    300


    TABLE  PROP(I,P)  proportion of available number of sorties for AC I

                   PER1   PER2   PER3   PER4
             AC1   0.60   0.50   0.70   0.70
             AC2   0.45   0.60   0.70   0.70


PARAMETER

**  Definition of WP numbers

      WP(K)   maximum number of WP k - 100000 represents infinity

                     / WP1       600
                       WP2    100000
                       WP3    100000
                       WP4    100000
                       WP5       600
                       WP6    100000
                       WP7    100000
                       WP8    100000
                       WP9    100000
                       WP10   100000
                       WP11   100000
                       WP12      600
                       WP15   100000
                       WP18   100000
                       WP19   100000
                       WP21   100000
                       WP22   100000
                       WP24   100000
                       WP25   100000
                       WP27   100000
                       WP34   100000
                       WP42   100000
                       WP45   100000
                       WP46      450   /


**  Definition of Weatherband Distribution

      PR(W)   probability of weatherband W
          /  WB1  0.00
             WB2  0.02
             WB3  0.14
             WB4  0.07
             WB5  0.07
             WB6  0.70     /


**  Parameter definition for Reconstitution

       TIME(P)  length of time period P
            / PER1  3
              PER2  4
              PER3  8
              PER4 15  /


        RECON(J)   number of days a killed target has to stay dead
```

```
                                  / TG5    3
                                    TG8   35
                                    TG10  20
                                    TG11   7
                                    TG12  35
                                    TG13  37
                                    TG14  40
                                    TG22  32
                                    TG24  30
                                    TG25   8
                                    TG27  30
                                    TG29  40
                                    TG34  34   /


        QTY(J)   maximum number of targets to be reconst. in 30 days

                                  / TG5    4
                                    TG8    2
                                    TG10  10
                                    TG11   2
                                    TG12   2
                                    TG13   2
                                    TG14   0
                                    TG22   7
                                    TG24   2
                                    TG25  20
                                    TG27   0
                                    TG29   0
                                    TG34   3   /


   PERUP(J,P) upper bound on time periods considered for reconstitution;

*  a killed target must exceed a minimum time > RECON(J) <  before it
*  is allowed to be reconstituted


    LOOP((J,P),

    PERUP(J,P) = SUM(PP$(ORD(PP) LE ORD(P)),1$(RECON(J) LT (SUM(PPP$

    ( (ORD(PPP) LE (ORD(P)+1)) AND (ORD(PPP) GE ORD(PP)) ),TIME(PPP))

                              - CEIL(0.5 * TIME(PP)) ) ) )         ) ;



*******************************************************
*                                                     *
*          Begin of aggregated INPUT DATA             *
*                                                     *
*******************************************************
```

```
TABLE  E(I,J,R)    Number of Targets type J killed by one Sortie type I


                       OD1       OD2       OD3       OD4
          AC1.TG5     .159      .156      .193      .310
          AC1.TG8     .305      .418      .299      .327
          AC1.TG10    .083      .120      .076      .276
          AC1.TG11    .081      .092      .077      .034
          AC1.TG12    .028      .010      .020      .044
          AC1.TG13    .216      .269      .205      .208
          AC1.TG14    .386      .328      .284      .292
          AC1.TG22    .343      .468      .333      .305
          AC1.TG24    .273      .232      .273      .218
          AC1.TG25    .134      .072      .067      .042
          AC1.TG27    .933      .913      .792      .741
          AC1.TG29    .137      .139      .092      .117
          AC1.TG34    .298      .172      .150      .428
          AC2.TG5     .247      .241      .288      .282
          AC2.TG8     .262      .305      .365      .418
          AC2.TG10    .083      .120      .076      .276
          AC2.TG11    .081      .092      .077      .034
          AC2.TG12    .028      .010      .020      .044
          AC2.TG13    .195      .216      .260      .269
          AC2.TG14    .685      .552      .569      .388
          AC2.TG22    .251      .343      .468      .350
          AC2.TG24    .205      .206      .273      .138
          AC2.TG25    .134      .072      .067      .042
          AC2.TG27    .652      .933      .913      .792
          AC2.TG29    .137      .064      .139      .092
          AC2.TG34    .382      .367      .338      .231



TABLE  B(I,J,R,W)  Weaponload Array for each set <i j r w>

                     OD1.WB1   OD1.WB2   OD1.WB3   OD1.WB4   OD1.WB5   OD1.WB6
        AC1.TG5        0         2         2         2         2         2
        AC1.TG8        0         2         2         2         2         2
        AC1.TG10       0         2         2         2         2         2
        AC1.TG11       0         2         2         2         2         2
        AC1.TG12       0         0         0         2         2         2
        AC1.TG13       0         2         2         2         2         2
        AC1.TG14       0         0         6         6         6         6
        AC1.TG22       0         2         2         2         2         2
        AC1.TG24       0         0         0         2         2         2
        AC1.TG25       0         2         2         2         2         2
        AC1.TG27       0         0         6         6         6         6
        AC1.TG29       0         0         6         6         6         6
        AC1.TG34       0         6         6         6         6         6
        AC2.TG5        0         6         6         6         6         6
        AC2.TG8        0         6         6         6         6         6
        AC2.TG10       0         2         2         2         2         2
        AC2.TG11       0         2         2         2         2         2
        AC2.TG12       0         0         0         2         2         2
        AC2.TG13       0         6         6         6         6         6
        AC2.TG14       0         0         4         4         4         4
```

```
          AC2.TG22        0       6       6       6       6       6
          AC2.TG24        0       6       6       6       6       6
          AC2.TG25        0       2       2       2       2       2
          AC2.TG27        0       0       0       0       0       6
          AC2.TG29        0       0       6       6       6       6
          AC2.TG34        0       0       4       4       4       4

     +               OD2.WB1 OD2.WB2 OD2.WB3 OD2.WB4 OD2.WB5 OD2.WB6
          AC1.TG5         0       2       2       2       2       2
          AC1.TG8         0       0       0       0       0       0
          AC1.TG10        0       0       0       0       0       0
          AC1.TG11        0       0       0       0       0       0
          AC1.TG12        0       0       0       0       0       0
          AC1.TG13        0       0       0       0       0       0
          AC1.TG14        0       0       0       0       0       0
          AC1.TG22        0       0       0       0       0       0
          AC1.TG24        0       0       6       6       6       6
          AC1.TG25        0       0       0       0       0       0
          AC1.TG27        0       0       0       0       0       0
          AC1.TG29        0       0       0       0       0       0
          AC1.TG34        0       0       0       2       2       2
          AC2.TG5         0       6       6       6       6       6
          AC2.TG8         0       2       2       2       2       2
          AC2.TG10        0       0       0       0       0       0
          AC2.TG11        0       0       0       0       0       0
          AC2.TG12        0       0       0       0       0       0
          AC2.TG13        0       2       2       2       2       2
          AC2.TG14        0       4       0       0       0       0
          AC2.TG22        0       2       2       2       2       2
          AC2.TG24        0       0       0       0       0       0
          AC2.TG25        0       0       0       0       0       0
          AC2.TG27        0       0       6       6       6       6
          AC2.TG29        0       0       0       6       6       6
          AC2.TG34        0       0       0       6       6       6

     +               OD3.WB1 OD3.WB2 OD3.WB3 OD3.WB4 OD3.WB5 OD3.WB6
          AC1.TG5         0       0       0       0       0       0
          AC1.TG8         0       2       2       2       2       2
          AC1.TG10        0       2       2       2       2       2
          AC1.TG11        0       2       2       2       2       2
          AC1.TG12        0       0       2       2       2       2
          AC1.TG13        0       2       2       2       2       2
          AC1.TG14        0       2       2       2       2       2
          AC1.TG22        0       2       2       2       2       2
          AC1.TG24        0       0       2       2       2       2
          AC1.TG25        0       0       0       0       0       0
          AC1.TG27        0       6       0       0       0       0
          AC1.TG29        0       0       2       2       2       2
          AC1.TG34        0       2       2       0       0       0
          AC2.TG5         0       0       0       0       0       0
          AC2.TG8         0       0       0       0       0       0
          AC2.TG10        0       2       2       2       2       2
          AC2.TG11        0       2       2       2       2       2
          AC2.TG12        0       0       2       2       2       2
          AC2.TG13        0       0       0       0       0       0
          AC2.TG14        0       0       0       0       0       0
```

```
      AC2.TG22         0        0        0        0        0        0
      AC2.TG24         0        0        0        2        2        2
      AC2.TG25         0        0        0        0        0        0
      AC2.TG27         0        0        0        0        0        0
      AC2.TG29         0        0        0        0        0        0
      AC2.TG34         0        6        6        0        0        0

   +                OD4.WB1  OD4.WB2  OD4.WB3  OD4.WB4  OD4.WB5  OD4.WB6
      AC1.TG5          0        6        6        6        6        6
      AC1.TG8          0        6        6        6        6        6
      AC1.TG10         0        0        2        2        2        2
      AC1.TG11         0        0        0        0        0        0
      AC1.TG12         0        0        0        0        0        0
      AC1.TG13         0        6        6        6        6        6
      AC1.TG14         0        6        0        0        0        0
      AC1.TG22         0        6        6        6        6        6
      AC1.TG24         0        0        0        0        0        0
      AC1.TG25         0        0        0        0        0        0
      AC1.TG27         0        0        2        2        2        2
      AC1.TG29         0        0        0        0        0        0
      AC1.TG34         0        0        0        0        0        0
      AC2.TG5          0        0        0        0        0        0
      AC2.TG8          0        0        0        0        0        0
      AC2.TG10         0        0        2        2        2        2
      AC2.TG11         0        0        0        0        0        0
      AC2.TG12         0        0        0        0        0        0
      AC2.TG13         0        0        0        0        0        0
      AC2.TG14         0        6        6        6        6        6
      AC2.TG22         0        0        0        0        0        0
      AC2.TG24         0        0        0        0        0        0
      AC2.TG25         0        0        0        0        0        0
      AC2.TG27         0        6        0        0        0        0
      AC2.TG29         0        0        2        2        2        2
      AC2.TG34         0        0        0        0        0        0



TABLE  WPTYPE(I,J,R)

* For each sortie-target combination the weapon type K of order R
* is given if it is possible to use this weapon

                       OD1      OD2      OD3      OD4
       AC1.TG5          5        6        5        4
       AC1.TG8          5        5        7        3
       AC1.TG10         5        5        7       18
       AC1.TG11         5        5        7        5
       AC1.TG12         5        5        7        5
       AC1.TG13         5        5        7        3
       AC1.TG14         3        3        5        3
       AC1.TG22         5        5        7        3
       AC1.TG24         5        3        7        3
       AC1.TG25        24       24       24       24
       AC1.TG27         3        3        3        7
       AC1.TG29         3        3        7        7
       AC1.TG34         3        5        5        3
       AC2.TG5         2        1        2        1
       AC2.TG8         1        5        1        5
       AC2.TG10        5        5        7       18
       AC2.TG11        5        5        7        5
       AC2.TG12        5        5        7        5
       AC2.TG13        1        5        1        5
       AC2.TG14       12       12       12        1
       AC2.TG22        1        5        5        1
       AC2.TG24        1        1        5        1
       AC2.TG25       24       24       24       24
       AC2.TG27        1        3        3        3
       AC2.TG29        3        1        3        7
       AC2.TG34       12        1        1        1

******************************************
*                                        *
*         End of INPUT DATA              *
*                                        *
******************************************



**  Definition of Sortie Variable

*   SX(I,J,R,W,P) describes the number of sorties type I assigned
*   to a target of type J carrying any weapon feasible for tactic R
*   and weatherband W and in time period P


    POSITIVE VARIABLES   SX(I,J,R,W,P)                               ;

**  Initial Values for Variables

            SX.L(I,J,R,W,P) = 0                                      ;

**  Declaration of variable EXPO(J,P)

    POSITIVE VARIABLE EXPO(J,P)                                      ;

**  Declaration of Kill Variable

    POSITIVE VARIABLE KILL(J,P)                                      ;

**  Declaration of Variable D(J,P)

    POSITIVE VARIABLE D(J,P)                                         ;

**  Declaration of Variable for cumulative weapon consumption

    POSITIVE VARIABLE WEAP(K)                                        ;

**  Upper bound for variable Weapon Consumption

            WEAP.UP(K)    = WP(K)                                    ;
```

```
**  Declaration of variable for number of targets been reconstituted

    POSITIVE VARIABLE REBUILD(J,P)                                   ;

**  Upper bound for variable REBUILD

            REBUILD.UP(J,P) = QTY(J) * TIME(P+1) / 30                ;



**  Variable definition for objective function

    VARIABLE  Z                                                      ;


EQUATIONS

    KILLVAL          maximize the value of destroyed targets
    KILLNL(J,P)      determines the number of killed targets
    EXPONENT(J,P)    evaluates the values of the exponential terms
    DEADTG(J,P)      determines the number of dead targets
    KILLCON(J,P)     constraint the number of killed targets
    RECCON(J,P)      constraint the max. number of targets for reconst.
    SORTCON(I,P)     constraint the number of allocated sorties
    WEAPCONSUM(K)    determines the consumption of each weapon type
    SELECT(I,J,R,W)  decides if next weapon on list can be used
    DISTR(I,J,W,P)   ensures that all weatherbands are covered prop. ;


KILLVAL..

            Z =E= SUM((J,P),V(J,P) * KILL(J,P))                      ;


KILLNL(J,P)..

     KILL(J,P) =E= ( (T(J)/C(J)) - D(J,P) ) * ( 1 - EXPO(J,P) )      ;


EXPONENT(J,P)..

     EXPO(J,P) =E= EXP( ((-C(J))/T(J)) * SUM((I,R,W)$B(I,J,R,W),

                              E(I,J,R) * SX(I,J,R,W,P)$B(I,J,R,W)) ) ;


DEADTG(J,P)..

   D(J,P) =E= SUM(PP$(ORD(PP) LT ORD(P)),KILL(J,PP) - REBUILD(J,PP)) ;


KILLCON(J,P)..

         KILL(J,P) =L= T(J) - D(J,P)                                 ;


```

```
RECCON(J,P)..

         SUM(PP$(ORD(PP) LE ORD(P)),REBUILD(J,PP)) =L=

                         SUM(PP$(ORD(PP) LE PERUP(J,P)),KILL(J,PP) ) ;


SORTCON(I,P)..

     SUM((J,R,W)$B(I,J,R,W),

                    SX(I,J,R,W,P)$B(I,J,R,W)) =L= PROP(I,P) * S(I,P) ;


WEAPCONSUM(K)..

  WEAP(K) =E= SUM((I,J,R,W,P)$( (ORD(K) EQ WPTYPE(I,J,R)) AND

          (B(I,J,R,W) NE 0) ),B(I,J,R,W) * SX(I,J,R,W,P)$B(I,J,R,W)) ;


  SELECT(I,J,R,W)$B(I,J,R,W)..

      0 =E= SUM(P,SX(I,J,R,W,P)$B(I,J,R,W)) *

            SUM((K,RP)$( (ORD(RP) LT ORD(R)) AND

                  (B(I,J,RP,W) NE 0) AND (ORD(K) EQ WPTYPE(I,J,RP)) ),

                                                 (WP(K) - WEAP(K)) ) ;


DISTR(I,J,W,P)$SUM(R,B(I,J,R,W))..

         SUM(R,SX(I,J,R,W,P)$B(I,J,R,W)) =E= PR(W) *

              SUM((R,WPP)$B(I,J,R,WPP),SX(I,J,R,WPP,P)$B(I,J,R,WPP)) ;



MODEL AIRATTACK  /ALL/                                               ;


*  Limit for number of iterations

OPTION ITERLIM = 1000 , LIMCOL = 0 , LIMROW = 0                      ;

OPTION SOLPRINT = OFF , SYSOUT = OFF                                 ;


SOLVE AIRATTACK USING NLP MAXIMIZING Z                               ;


**   The following statements represent the solution values          ;

PARAMETERS
```

```
613
614        KILLTG(J,P)     number of targets J killed in period P
615        OBJECTIVE(P)       Objective Function Value
616        KILLPOT(J,P)       potential Kill-Capability (target-type vs period)
617        OPSORTIE(I,J,R,P,W) number of optimal sorties
618        SORTIE(J,P,I)   number of sorties I assigned to target J in period P
619        WPCOMB(I,J,K)   number of weapons (sortie , target  and weapon type)
620        WPCONS(R,K)        number of weapons (tactic vs weapon-type)
621        WEAPON(J,K)        number of weapons (target  vs weapon-type)   ;
622
623
624            KILLTG(J,P)  = KILL.L(J,P)                                   ;
625
626            OBJECTIVE(P) = SUM(J,V(J,P) * KILL.L(J,P))                   ;
627
628            KILLPOT(J,P) = SUM((I,R,W)$B(I,J,R,W),
629
630                                 E(I,J,R) * SX.L(I,J,R,W,P))             ;
631
632            WEAPON(J,K) = SUM((I,R,W,P)$(ORD(K) EQ WPTYPE(I,J,R)),
633
634                                         B(I,J,R,W) * SX.L(I,J,R,W,P)) ;
635
636            WPCONS(R,K) = SUM((I,J,W,P)$(
637
638            (ORD(K) EQ WPTYPE(I,J,R)) AND (B(I,J,R,W) NE 0) ),
639
640                                       B(I,J,R,W) * SX.L(I,J,R,W,P))   ;
641
642              OPSORTIE(I,J,R,P,W) = SX.L(I,J,R,W,P)                      ;
643
644              SORTIE(J,P,I)      = SUM((R,W),SX.L(I,J,R,W,P))            ;
645
646
647
648   OPTION OBJECTIVE:2     ; DISPLAY OBJECTIVE   ;
649   OPTION KILLTG:1:1:1    ; DISPLAY KILLTG      ;
650   OPTION KILLPOT:1:1:1   ; DISPLAY KILLPOT     ;
651   OPTION OPSORTIE:1:2:1  ; DISPLAY OPSORTIE    ;
652   OPTION SORTIE:1:1:2    ; DISPLAY SORTIE      ;
653   OPTION WPCONS:1:1:1    ; DISPLAY WPCONS      ;
654   OPTION WEAPON:1:1:1    ; DISPLAY WEAPON      ;
655   OPTION WEAP:1          ; DISPLAY WEAP.L      ;
656   OPTION REBUILD:1:1:1   ; DISPLAY REBUILD.L   ;


COMPILATION TIME     =         2.140 SECONDS


MODEL STATISTICS    SOLVE AIRATTACK USING NLP FROM LINE 607




MODEL STATISTICS
```

```
BLOCKS OF EQUATIONS    10      SINGLE EQUATIONS   932
BLOCKS OF VARIABLES     7      SINGLE VARIABLES  1289
NON ZERO ELEMENTS    9758      NON LINEAR N-Z    1889
DERIVATIVE POOL        31      CONSTANT POOL       61
CODE LENGTH         15943

GENERATION TIME      =       65.580 SECONDS


EXECUTION TIME       =       67.680 SECONDS


SOLUTION REPORT     SOLVE AIRATTACK USING NLP FROM LINE 607


               S O L V E     S U M M A R Y

     MODEL   AIRATTACK            OBJECTIVE  Z
     TYPE    NLP                  DIRECTION  MAXIMIZE
     SOLVER  MINOS5               FROM LINE  607

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      2 LOCALLY OPTIMAL
**** OBJECTIVE VALUE               1358.0172

 RESOURCE USAGE, LIMIT          64.179      1000.000
 ITERATION COUNT, LIMIT       639           1000
 EVALUATION ERRORS              0              0

     M I N O S  ---  VERSION  5.0   APR  1984
     = = = = =
     COURTESY OF B. A. MURTAGH AND M. A. SAUNDERS,
                 DEPARTMENT OF OPERATIONS RESEARCH,
                 STANFORD UNIVERSITY,
                 STANFORD   CALIFORNIA   94305   U.S.A.


 WORK SPACE NEEDED (ESTIMATE)  --  104191 WORDS.
 WORK SPACE AVAILABLE          --  134740 WORDS.
 (MAXIMUM OBTAINABLE           --  288878 WORDS.)

 EXIT -- OPTIMAL SOLUTION FOUND
 MAJOR ITERATIONS           22
 NORM RG / NORM PI  5.752E-08
 TOTAL USED             65.17 UNITS
 MINOS5 TIME            56.27 (INTERPRETER -       9.78)


**** REPORT SUMMARY :          0     NONOPT
                               0 INFEASIBLE
                               0  UNBOUNDED
                               0     ERRORS
```

```
----     648 PARAMETER OBJECTIVE     OBJECTIVE FUNCTION VALUE

PER1 251.30,    PER2 333.84,    PER3 674.04,    PER4  98.84

----     649 PARAMETER KILLTG        NUMBER OF TARGETS J KILLED IN PERIOD P

               PER1        PER2        PER3        PER4

TG5             0.5         1.1        18.0         2.0
TG8            13.0
TG10                                               26.3
TG11            9.6
TG13            2.2                     1.7         0.1
TG14            2.0
TG22                                                6.0
TG24                                    1.5         1.3
TG25                       26.6         1.1
TG27                                   21.0
TG34                                   18.0

----     650 PARAMETER KILLPOT       POTENTIAL KILL-CAPABILITY (TARGET-TYPE VS
                                     PERIOD)

               PER1        PER2        PER3        PER4

TG5             0.5         1.1        20.1         2.5
TG8            13.7
TG10                                               29.0
TG11            9.8
TG13            2.4                     2.2         0.2
TG14            2.1
TG22                                                6.7
TG24                                    1.9         3.3
TG25                       28.9         1.3
TG27                                   36.1
TG34                                   20.1

----     651 PARAMETER OPSORTIE      NUMBER OF OPTIMAL SORTIES

INDEX 1 = AC1 INDEX 2 = TG8

                    WB2         WB3         WB4         WB5         WB6

OD3.PER1            0.2         1.2         0.6         0.6         5.8

INDEX 1 = AC1 INDEX 2 = TG10

                    WB2         WB3         WB4         WB5         WB6

OD1.PER4            4.2        29.4        14.7        14.7        99.9
OD3.PER4                                                           47.1

INDEX 1 = AC1 INDEX 2 = TG11
```

```
                  WB2         WB3         WB4         WB5         WB6

OD1.PER1          2.0        14.0         7.0         7.0        69.8

INDEX 1 = AC1 INDEX 2 = TG13

                  WB2         WB3         WB4         WB5         WB6

OD1.PER3          0.2         1.4         0.7         0.7         7.2

INDEX 1 = AC1 INDEX 2 = TG25

                  WB2         WB3         WB4         WB5         WB6

OD1.PER2          2.0        14.0         7.0         7.0        70.0
OD1.PER3          0.2         1.3         0.7         0.7         6.5

INDEX 1 = AC1 INDEX 2 = TG27

                  WB2         WB3         WB4         WB5         WB6

OD1.PER3                      5.4         2.7         2.7        27.2
OD3.PER3          0.8

INDEX 1 = AC1 INDEX 2 = TG34

                  WB2         WB3         WB4         WB5         WB6

OD1.PER3          0.9         6.5         3.3         3.3        32.5

INDEX 1 = AC2 INDEX 2 = TG5

                  WB2         WB3         WB4         WB5         WB6

OD1.PER1    4.3313E-2         0.3         0.2         0.2         1.5
OD1.PER2    8.6886E-2         0.6         0.3         0.3         3.0
OD1.PER3          1.6        11.4         5.7         5.7        56.9
OD1.PER4          0.2         1.4         0.7         0.7         7.0

INDEX 1 = AC2 INDEX 2 = TG8

                  WB2         WB3         WB4         WB5         WB6

OD1.PER1          0.9         6.0         3.0         3.0        29.9

INDEX 1 = AC2 INDEX 2 = TG10

                  WB2         WB3         WB4         WB5         WB6

OD3.PER4          3.1        21.9        10.9        10.9       109.4

INDEX 1 = AC2 INDEX 2 = TG11

                  WB2         WB3         WB4         WB5         WB6
```

```
OD1.PER1         0.4         2.9         1.5         1.5        14.6

INDEX 1 = AC2 INDEX 2 = TG13

                 WB2         WB3         WB4         WB5         WB6

OD1.PER1         0.2         1.7         0.8         0.8         8.4
OD1.PER4  1.7758E-2          0.1   6.2153E-2   6.2153E-2         0.6

INDEX 1 = AC2 INDEX 2 = TG14

                 WB2         WB3         WB4         WB5         WB6

OD1.PER1                     0.4         0.2         0.2         2.2
OD2.PER1   6.1764E-2

INDEX 1 = AC2 INDEX 2 = TG22

                 WB2         WB3         WB4         WB5         WB6

OD1.PER4         0.5         3.7         1.9         1.9        18.7

INDEX 1 = AC2 INDEX 2 = TG24

                 WB2         WB3         WB4         WB5         WB6

OD1.PER3         0.1         1.0
OD1.PER4         0.3         2.3         1.1         1.1        11.3
OD3.PER3                                 0.5         0.5         5.1

INDEX 1 = AC2 INDEX 2 = TG25

                 WB2         WB3         WB4         WB5         WB6

OD1.PER2         2.3        16.2         8.1         8.1        81.0

INDEX 1 = AC2 INDEX 2 = TG34

                 WB2         WB3         WB4         WB5         WB6

OD1.PER3                     2.3         1.1         1.1        11.4
OD3.PER3         0.3


----    652 PARAMETER SORTIE        NUMBER OF SORTIES I ASSIGNED TO TARGET J
                                    IN PERIOD P

        PER1.AC1    PER1.AC2    PER2.AC1    PER2.AC2    PER3.AC1    PER3.AC2

TG5                      2.2                     4.3                    81.3
TG8          8.3        42.8
TG11        99.7        20.9
TG13                    12.1                                10.3
TG14                     3.1
TG24                                                                     7.4
TG25                               100.0       115.7         9.3
```

```
TG27                                                        38.8
TG34                                                        46.5        16.3

   +    PER4.AC1    PER4.AC2

TG5                     10.0
TG10       210.0       156.3
TG13                     0.9
TG22                    26.7
TG24                    16.1

----    653 PARAMETER WPCONS        NUMBER OF WEAPONS (TACTIC VS WEAPON-TYPE)

            WP1         WP2         WP3         WP5         WP7        WP12

OD1       598.0       586.8       507.3       587.6                    76.2
OD2                                                                     0.2
OD3          2.0                    4.7        12.4       423.5

  +        WP46

OD1       450.0

----    654 PARAMETER WEAPON        NUMBER OF WEAPONS (TARGET  VS WEAPON-TYPE)

             WP1         WP2         WP3         WP5         WP7        WP12

TG5                    586.8
TG8        256.6                                            16.7
TG10                                           325.8       406.9
TG11                                           241.2
TG13        77.7                                20.7
TG14                                                                    12.4
TG22       160.0
TG24       103.7                                12.4
TG27                               233.0
TG34         2.0                   279.0                                64.1

   +        WP46

TG25       450.0

----    655 VARIABLE  WEAP.L

WP1  600.0,    WP2  586.8,    WP3  511.9,    WP5  600.0,    WP7  423.5
WP12  76.4,    WP46 450.0

----    656 VARIABLE  REBUILD.L

            PER1        PER2        PER3

TG5          0.5         1.1         2.0
```

```
TG11                        0.5          1.0
TG25                        5.3         10.0


EXECUTION TIME       =         22.580 SECONDS
```

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